Local Nonlinear Problems in Beam Structures Made of Plastics

MARIUS-ALEXANDRU GROZEA[1], ANTON HADAR[1]*
[1]Politehnica University of Bucharest – Strength of Materials Department, 313, Spl. Independentei, 060042, Bucharest, Romania

This work brings to the forefront local nonlinear problems that appear in junctions of beam structures made of plastics, subjected to heavy loads. We also revealed the notions of local problems and heavy loads. These theoretical aspects are explained with the help of two beam structures of various complexity, made of plastic, with local problem. The results are presented as stress maps, maps of mechanical strains, charts with the evolution of the states reached in the structure with the loading steps. We mentioned the situations that require local study; nonlinear problems classification is presented and the methodology for nonlinear calculus is described.

Keywords: local problem, beam structure, heavy loads, nonlinearity

In mechanical engineering there is a wide range of machinery, plants, equipment, devices etc., which are composed of parts with different geometries, from simple to complex, which ensure strength, stability, security, accuracy, durability etc. and are called mechanical structures.

The mechanical structure is standard defined as a complex system, rigorously operationally, geometrically and mechanically defined, consisting of individual mechanical parts.The collapse is a problem with a pronounced local character for all kinds of mechanical structures, whatever their structural, functional and implementation technologies characteristics are. The collapse is initiated at a point or a small region (compared with the dimensions of the structure) and then expands and spreads until the structure loses its ability to accomplish the functional role for which it was designed. Therefore, the local analysis is very important.

The stress state is considered to be local when the intensity is relatively high in a small area in relation to the structure gauge.

In some cases, when the structure has high importance for human safety or to prevent damage, it may become necessary to analyze the local stress, i.e. performing calculus at a different scale. Thus, the study makes the transition from global to local study. Modeling and analysis of local problems differ from those used in fracture mechanics: the local problems study is "macro" scale and cracks in fracture mechanics are "micro" scale.

The cases where there is need for detailed calculations are:

-behaviour under variable loads (not very high local stresses can lead to structural failure by fatigue, as a result of variable loads);

-endurance effects;

-determination of local stress peaks with very high values, exceeding the limit of proportionality;

-situations when the material is fragile, or if the operating of the structure can cause fragilization of the material, the local analysis is mandatory in the area in which the collapse appears to initiate.

So, it comes to the nonlinear calculus.

Non-linearity

The finite element method is an excellent tool for the analysis of structures with non-linear behaviour.

* email: antonhadar@yahoo.com

In the practice of FE modeling and analysis, the following classification may be considered [1]:

- problems with physical non-linearity (of the material). Here, Hooke's law is replaced with a more complicated stress – strain law, depending on the shape of the characteristic curve of the material.

- problems with geometric non-linearity. In this category, problems where elevated deformations are produced due to the loads are considered. The linear elastic behaviour is considered, but the relations between strains and displacements and between loads and displacements are non-linear. Also, the efforts depend on displacements and the equilibrium equations written for the undeformed structure do not remain valid for the deformed structure.

- problems with general non-linearity. This is the case when physical and geometric non-linearity is superimposed.

Solving methods

The incremental method

This is also known as step-by-step method . The load is divided in several smaller load steps or increments. Usually, these increments are equal, but they may be unequal too. The load grows with each step; during each increment, a linear elastic behaviour (a constant [K] matrix) is assumed to occur. On the next step, this matrix changes. The solution for each step "*i*", with a growth DF_i of the load, is obtained as a displacement increment Δu_i. These „growths" of the displacements are cumulated in order to obtain the total displacement for each „stage" of loading. The process continues until the entire load is applied. The scheme is presented in figure 1 [1, 7].

The calculus relations are written by taking into account the initial state of the structure, defined by the initial loads $\{F_0\}$ and initial displacements $\{u_0\}$. Usually, these vectors are null, because the structure is neither loaded nor deformed. An initial equilibrium state may be defined in the case when initial loads or displacements exist.

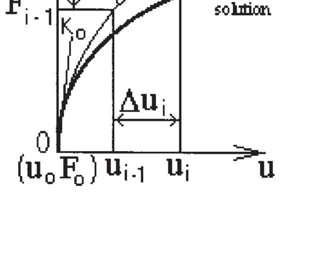
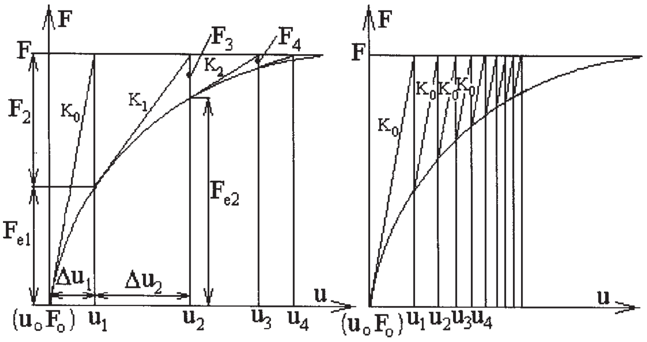


Fig. 1. Incremental solution



a b

Fig. 2

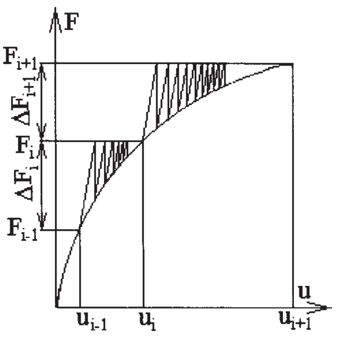


Fig. 3

If the total load is divided in m steps, then the total effective load is:

$$\{F\} = \{F_0\} + \Sigma \{\Delta F_j\}, \ j = 1, \ldots, m,$$

where Δ represents a finite increment. After applying the i-th increment, the load is:

$$\{F_i\} = \{F_0\} + \Sigma \{\Delta F_j\}, \ j = 1, \ldots, i,$$

in which $\{F_m\} = \{F\}$. A similar procedure is used for the displacements. Thus:

$$\{u_i\} = \{u_0\} + \Sigma \{\Delta u_j\}, \ j = 1, \ldots, i. \qquad (1)$$

For calculating the displacement increment, the value of the stiffness matrix $[K_{i-1}]$ at the end of the previous step is used, that means:

$$[K_{i-1}] \{\Delta u_i\} = \{\Delta F_i\}, \ i = 1, 2, 3, \ldots, m,$$

where

$$[K_{i-1}] = [K_{i-1} (\{u_{i-1}\}, \{F_{i-1}\})],$$

and $[K_0]$ is the initial stiffness matrix, calculated for the initial geometric configuration of the structure and for the material constants determined from the characteristic curve at the beginning of the loading process [1, 7].

The iterative method

In this case, the structure is loaded with the entire load in each iteration. Because an approximate, constant value of the stiffness is considered for each iteration, the equilibrium conditions are not fulfilled. After each iteration, the part of the total load that does not satisfy these conditions is calculated. This part is used in the next iteration, in order to determine an additional increase of the displacements. The process is repeated until the equilibrium equations are acceptably satisfied. Essentially, the iterative method consists of successive corrections of the solution until the equilibrium equations under the total load $\{F\}$ are fulfilled.

If in the general case initial loads $\{F_0\}$ and displacements $\{u_0\}$ exist, then the load for the i-th cycle of the iterative process is calculated with the relation:

$$\{F_i\} = \{F\} - \{F_e, i\text{-}1\},$$

where $\{F\}$ is the total load and $\{F_{e,i-1}\}$ is the load in equilibrium after the previous iteration. The increase of the displacements, calculated for the i-th step is:

$$[K_{(i)}] \{\Delta u_i\} = \{F_i\}. \qquad (2)$$

The total displacement after the i-th iteration is calculated with relation (2). Finally, the load $\{F_{e,i}\}$, necessary for maintaining the displacements $\{u_i\}$ is determined.

The iterative process is continued until the increase of displacements or the forces that are not in equilibrium become zero, that means $\{\Delta u_i\}$ or $\{F_i\}$ become null or small enough.

About the calculus of the stiffness matrix $[K_{(i)}]$ from equation (2), usually this matrix is determined for the previous step, in the point $\{u_{i-1}\}$, $\{F_{i-1}\}$, this means $[K_{(i)}] = [K_{(i-1)}]$. It must be take into account that $[K_{(0)}]$ is the stiffness matrix for the initial state of the structure, defined by the values $\{F_0$ and $\{u_0\}$.

There are different variants for the iterative method, differing by the way of considering the stiffness matrix of the structure $[K]$. In figure 2,a one shows the scheme of the basic iterative method and in figure 2.b – a modified variant, that uses the initial value $[K_{(0)}]$ for all iterations. In the last case, a greater number of iterations is necessary, but a greater speed of the calculus process is achieved, because there is no need to recalculate the matrix $[K]$ in each iteration. The iterative method is similar to the numerical procedures for solving non-linear equations, like Newton or Newton – Raphson methods [1, 7].

The mixed method.

It is also called iterative in steps and represents a combination between the iterative and the incremental method. The scheme of the mixed method is shown in figure 3. In this method, the load is applied incrementally, and successive iterations are performed after each increment. The mixed method is more efficient, but requires a greater volume of calculations [1, 7].

Methodology of calculus

In order to perform a nonlinear analysis one must follow the next steps:

-selecting a suitable finite element which will be used for meshing; element choice is made depending on the type and geometry of the structure, as well as its available options for post-processing the results;

-entry the material in the database of the program; the material is usually a structural one, with nonlinear behavior, “inelastic”. The characteristics E and ν are introduced, then the curve is defined by a sequence of pairs (ε, σ);

-modeling of the structure is done;

-the structure is meshed; this process is done so in areas of stress concentrators, network density is sufficiently high;

-displacement and load definition is made;

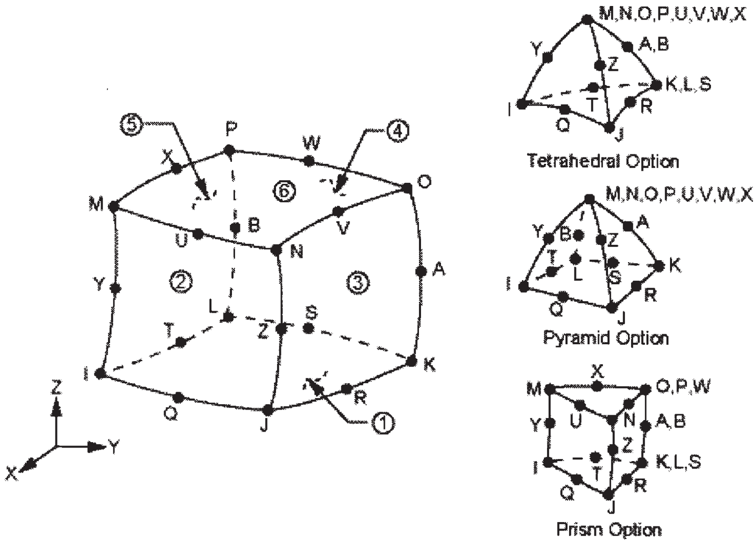


Fig. 4. SOLID type finite element with 20 knots

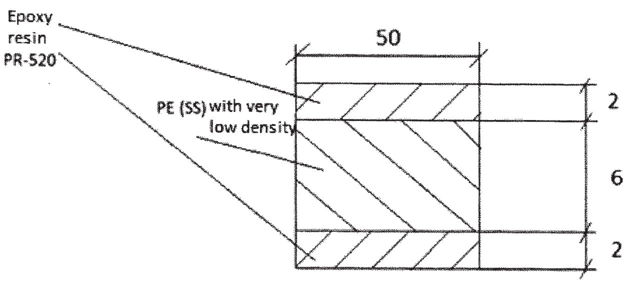


Fig. 5. Sections of the beams used

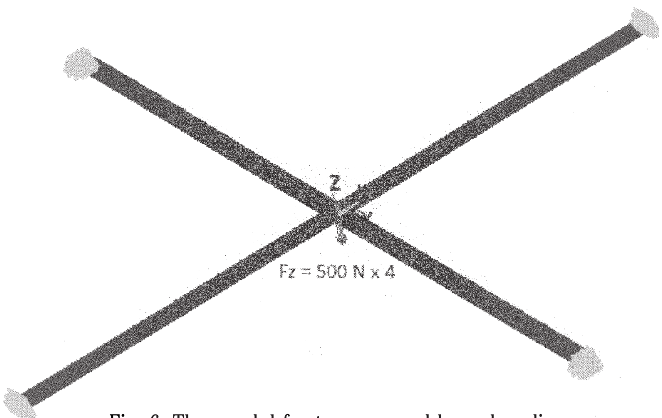


Fig. 6. The model for two crossed bars, bending

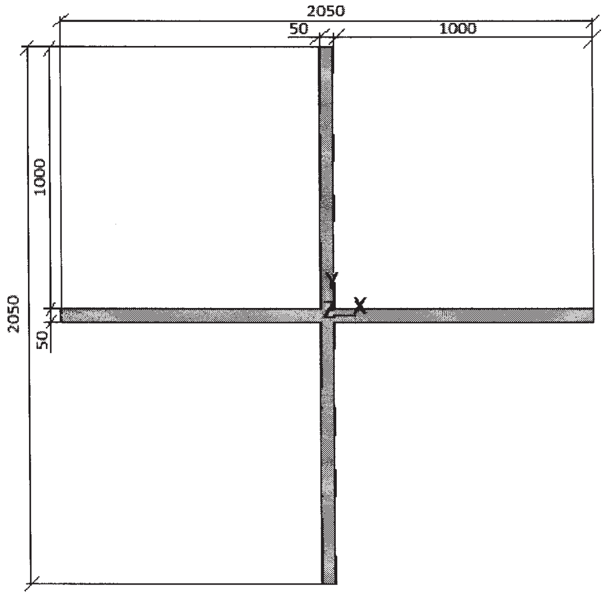


Fig. 7. Gauge of the analyzed structure

-the specific data for nonlinear analysis is introduced - the number of loading steps (the minimum and maximum), one can then store the final and partial solutions;

-the analysis is performed;

-results are studied with various post-processing facilities, such as stress maps, displacement maps, charts etc [6].

Numerical Examples

To illustrate the theoretical concepts presented in the paper, there were chosen two structures of different complexity, whose modeling was done with solid-type finite elements with 20 nodes, having three degrees of freedom per node: translations in the nodal x, y, and z directions, as in figure 4.

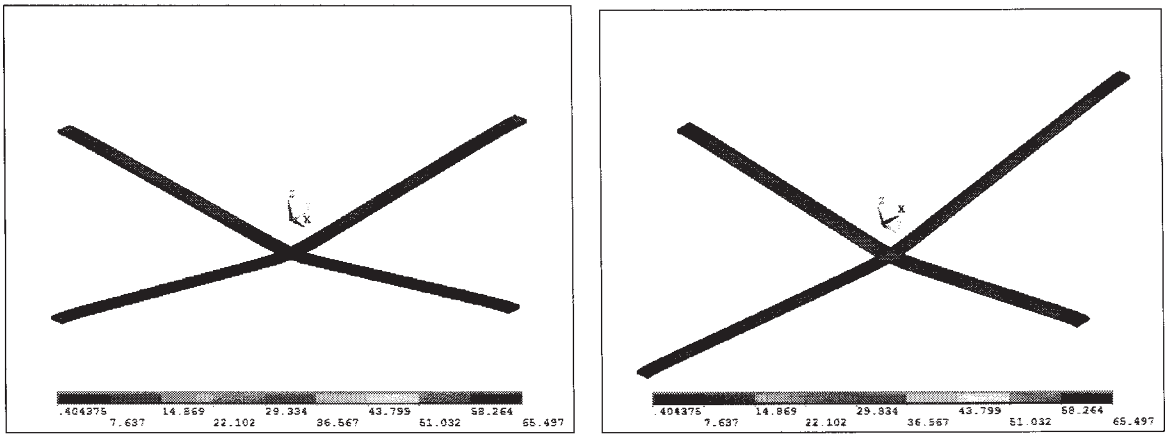
The first example chosen to illustrate the need of local analysis of the stress and strain state in a structure, consists of two perpendicular straight beams of rectangular section, one of the most common cases in engineering practice. The beams are made of a layered composite, consisting of two layers of PR-520 epoxy resin (E = 2000 MPa, ν = 0,.45) on the outside and the middle is made of PE (SS) with very low density (E = 172.37 MPa, ν = 0.3); the bar section is presented in figure 5.

In the analysis no account was taken of the adhesive layers between components. All four legs are embedded, and on the junction of the beams four concentrated loads of 500 N each are applied, on the vertical direction, as shown in the model in figure 6. Loading was applied in 10 steps.

Global equivalent von Mises stress state reached in the structure is currently being viewed from different positions in figure 8.a, b. In figure 8.b one can distinguish local effects produced by application of the load.

Studying the characteristic curves of the two plastics shown in figure 9, the structure is composed of and looking at the detail in figure 10, we conclude that in the core consisted of EP (SS), the stress state is minimal, and in the tough outer layers consisting of PR-520 epoxy resin, the stress values tend to reach the breaking limit.

A structure has a smaller or a larger number of local problems. Each of them is unique often even for the same structure and requires a proper modeling and analysis. For example, in figure 10, both in area 1 and area 2, the structure will achieve maximum stress (65 MPa), but area 1



a. b.

Fig. 8. Global status equivalent von Mises stress [MPa]: a. top view; b. view from underneath

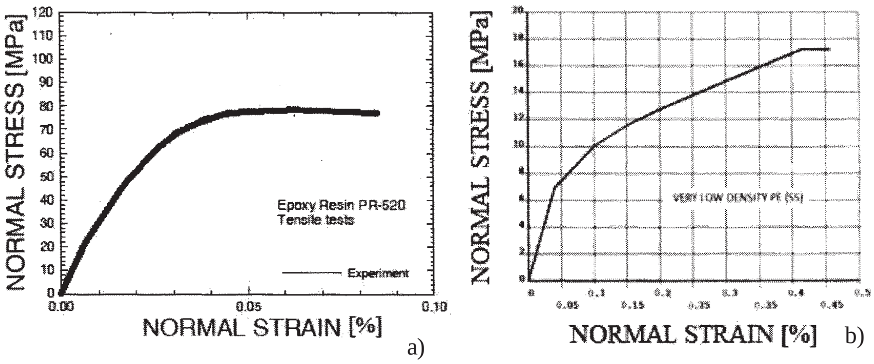


Fig. 9. Characteristic curves of plastic materials used:
a. PR-520 epoxy resin; b. EP (SS) with very low density

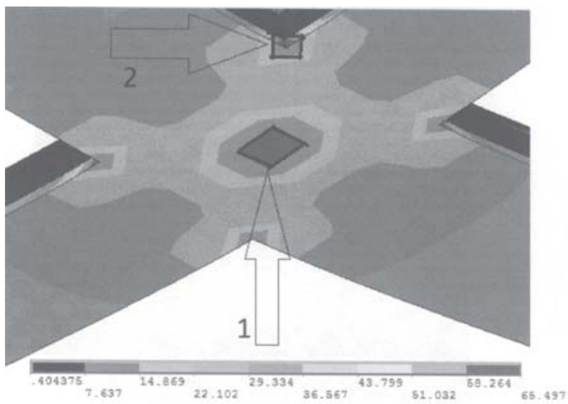


Fig. 10. Local von Mises stress state reached in the structure in the junction area [MPa]

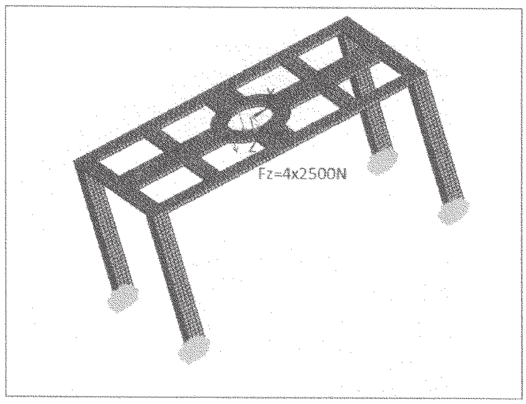


Fig. 12. The complex beam structure

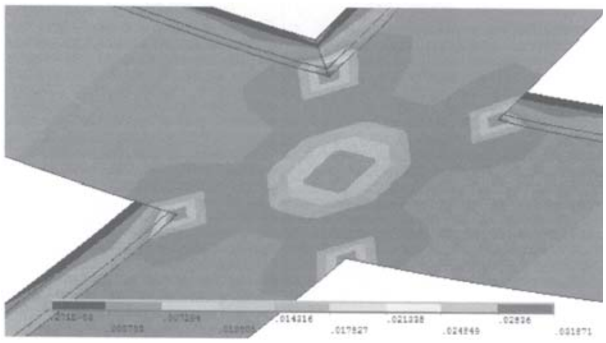


Fig. 11. Local specific strains state reached in the junction area [%]

represents a deterioration of the material surface as a result of the application of loads method and in area 2, the recorded values of the stress are due to the concentrator (90° junction). The problem in area 1 may be eliminated by distribuiting the load on a larger area and the one in area 2 by changing the local design of the junction from corner to radius.

Figure 11 presents a detail with the mechanical strain corresponding to the junction area illustrated in figure 10.

Another example of beam structure, made from PR-520 epoxy resin ($E = 2000$ MPa, $\nu = 0.45$), with a complex configuration is modeled in figure 12. The structure is a table that has the embedded and the load consists of four concentrated forces acting in the vertical direction each with 2500N. Loading was applied in 10 steps. The gauge of the structure is 2000 x 1000 x 1025 mm, corresponding to the three directions - X, Y, Z.

If it would not be taken into account the real curve of the material and E would be considered constant, the map of the stress state existing in the structure, according to the appropriate application of the last step, is shown in figure 13.

The real global equivalent von Mises stress state in the structure is shown in figure 14. It is found that the real

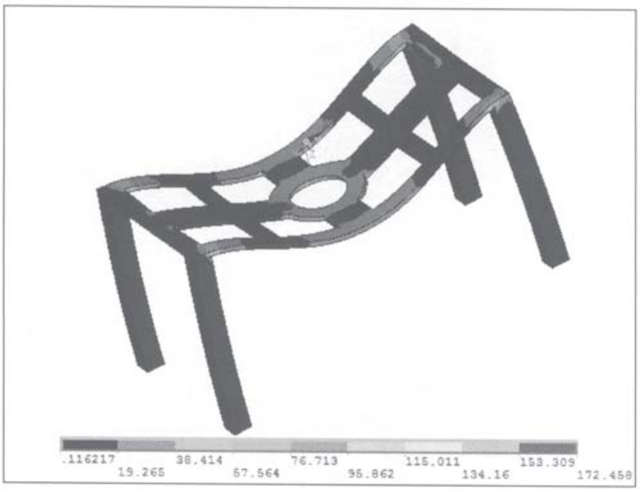


Fig. 13. Map of the hypothetical equivalent von Mises stress reached in the complex beam structure [MPa] if E should be constant

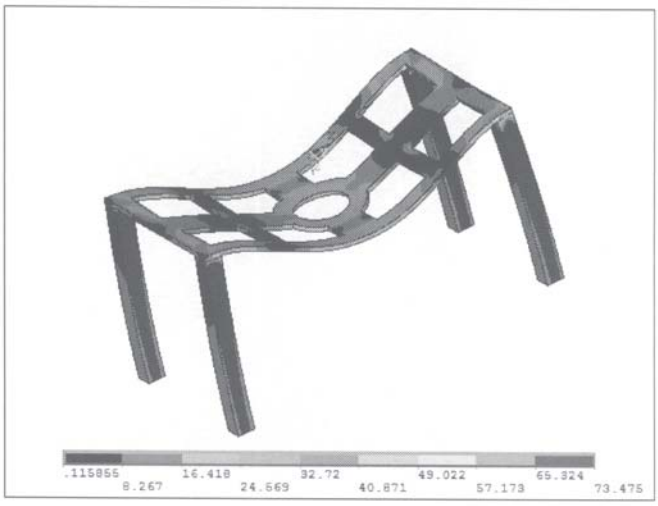


Fig. 14. Map of the equivalent von Mises stress reached in the complex beam structure [MPa]

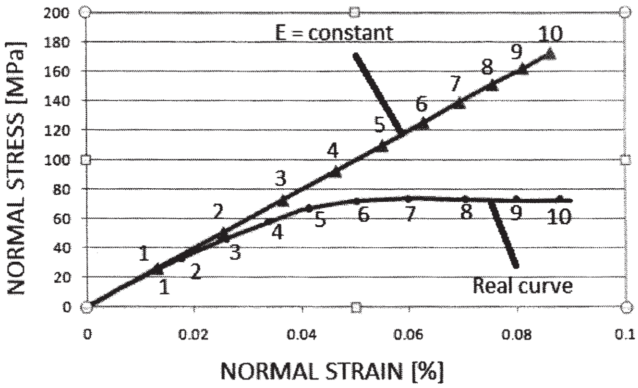
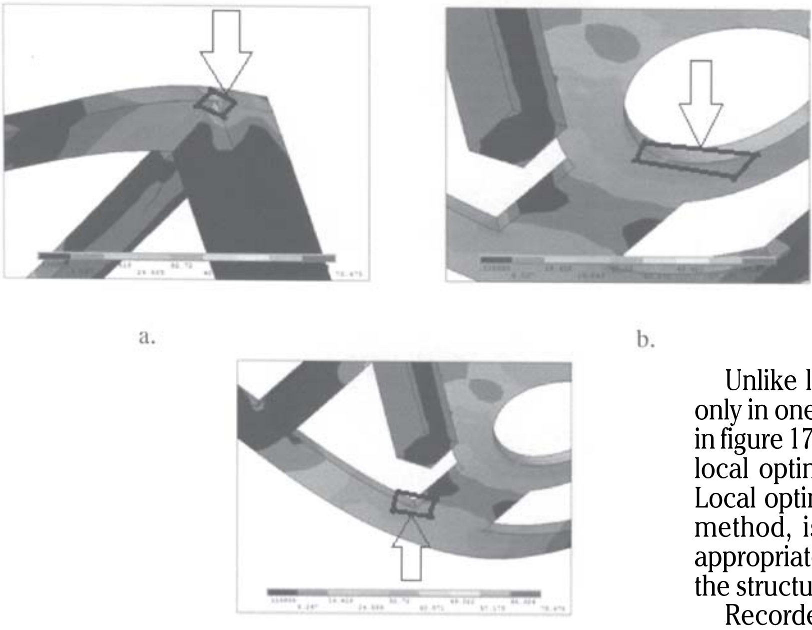


Fig. 15. Comparison between the states reached in the structure with the loading: when E is considered constant and when the the real curve is taken into consideration



a.

b.

c.

Fig. 16. Various local nonlinear problems arising in the same structure

maximum equivalent stress reached is 73 MPa, not 172 MPa as in the case when E it is taken as constant.

In figure 15, there is presented a comparison between the states reached in the structure, with each of the 10 loading steps for: the situation where E is constant and when the real curve is taken into account.

It may be noted that each junction of the complex structure is a particular case of local problems, as shown in figure 16.

Unlike local stress, specific strain reaches high values only in one type of junctions of the structure, as illustrated in figure 17. Thus, it requires a redesign of the structure and local optimization to eliminate the identified problems. Local optimization can be done using the substructuring method, isolating the affected area and imposing the appropriate conditions, taken from the global analysis of the structure, previously performed.

Recorded displacements are very large compared with the gauge of the structure, which makes the nonlinearity to be mixed. These displacements are shown in figure 18. If we look at the displacements obtained, we conclude that we have a structure with very large displacements compared to the gauge, so from this point of view redesign, followed by a new analysis is needed.

Failure and fracture mechanisms for plastics in general and in particular for the laminated composites are different and more complex than for the ordinary homogeneous and isotropic materials.

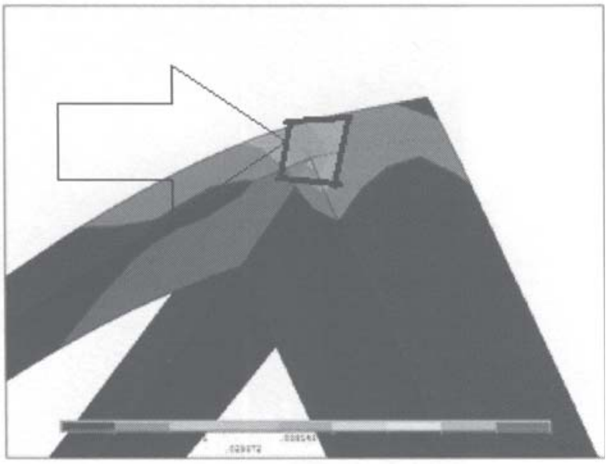


Fig. 17. Local strain state that occurs in only one type of structure areas

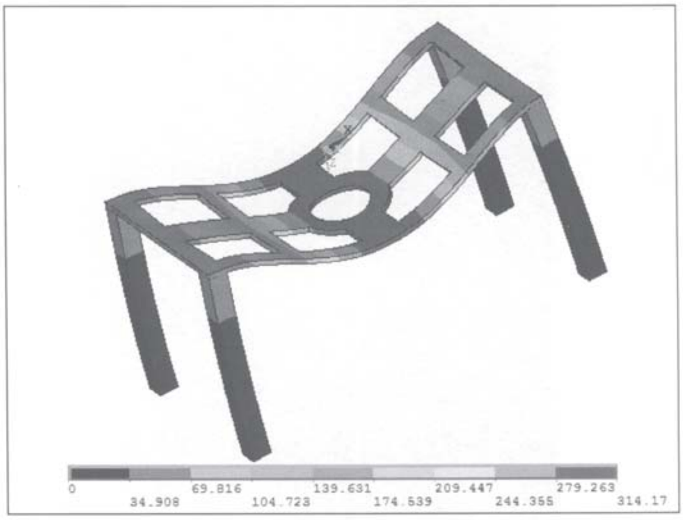


Fig. 18. The global state of displacements of the structure

For these reasons, modeling and analysis by calculus of the structures made of laminate composites and other plastics should be made primarily aiming to the determining of the local stress states in areas where high stress gradients, such as: blockings, applying points of the loads, geometrical or other discontinuities, concentrators, junctions etc. occur.

Conclusions

This paper presents both theoretical and practical elements.

The theoretical part contains:

- the definitions of the local problems, nonlinearity and local stress state;
- the explanation of the failure concept;
- the methodology that we developed for nonlinear calculus.

Analyzing the numerical results, we conclude that:

- detection of local problems is performed in order to optimize the structure so that the material and structure remain in the field of the use prescribed in the design phase. We also propose the solutions for eliminating the problems as follows: when we have geometrical nonlinearity, we make a global optimization of the structure; when we have material nonlinearity, we redesign the local areas or we optimize them using the substructuring method;
- it was proven that considering the real curve of the material, for the same load, we obtain lower stress and higher specific strains;
- each local problem is unique and often even for the same structure;
- the collapse is a problem with a pronounced local character for all kinds of mechanical structures, whatever their structural, functional and implementation technologies characteristics are.

References

1. CONSTANTINESCU, I.N., SOROHAN, ŞT., PASTRAMA, ŞT., The Practice of Finite Element Modeling and Analysis – Ed PRINTECH, Bucureşti, 2006

2. COOK, R.D., Finite Element Modeling for Stress Analysis, John Wiley &Sons, U.S.A., 1995

3. MATA, P., OLLER, S., BARBAT, A.H., Static Analysis of Beam Structures under Nonlinear Geometric and Constitutive Behavior, www.sciencedirect.com, 2007

4. CRISFIELD, M.A., Non-Linear Finite Element Analysis of Solids and Structures, vols. 1 & 2, John Wiley & Sons, 1998

5. HADĂR, A., CONSTANTINESCU, I.N., GHEORGHIU, H., COTEŢ, C.E, Modelare şi modele pentru calcule în ingineria mecanică, Ed PRINTECH, Bucureşti, 2007

6. GROZEA M.-A., HADĂR, A., Methodology for Nonlinear Numerical Calculus of Structures, Analele Universităţii din Oradea, Fascicula „Construcţii şi Instalaţii Hidroedilitare", november 2010

7. CONSTANTINESCU, I.N., PICU, R.C., HADAR, A., GHEORGHIU, H., Rezistenta materialelor pentru ingineria mecanica, Ed BREN, Bucuresti, 2006.

8. ZGÎRIAN, G., DEMETRESCU, I., GHEORGHIU, H., IOVU, H., HADAR, A., ATANASIU, C., Rev. Chim. (Bucharest), **56**, no. 7, 2005, p. 757

9. HADAR, A., NICA, M.N., CONSTANTINESCU, I.N., PASTRAMA, ST., The Constructive and Geometric Optimization of the Junctions in the Structures Made from Laminated Composite Materials, STROJINSKI VESTNIK – JOURNAL OF MECHANICAL ENGINEERING, 52, no. 7-8, Jul-Aug. 2006, p. 546

Manuscript received: 16.05.2011